Eur. Phys. J. A **24**, 167–172 (2005)
DOI 10.1140/epja/i2005-10005-7

THE EUROPEAN PHYSICAL JOURNAL A

Evidence for wobbling excitation in ^{161}Lu

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Received: 21 January 2005 / Revised version: 14 February 2005 /

Communicated by D. Schwalm

Abstract. High-spin states in ^{161}Lu were investigated using the EUROBALL spectrometer. A previously known triaxial superdeformed band has been extended to higher spins and a new band with similar characteristics has been discovered. Comparison to systematically occurring wobbling bands in Lu isotopes strongly suggests that these two bands represent the $n_{\mathrm{w}} = 0$ and 1 wobbling excitations in ^{161}Lu.

PACS. 21.10.-k Properties of nuclei; nuclear energy levels – 23.20.Lv γ transitions and level energies – 25.70.-z Low and intermediate energy heavy-ion reactions – 27.70.+q $150 \leq A \leq 189$

1 Introduction

Rotational-band spectra in the decay of excited nuclei were already observed more than 50 years ago. Bohr [1] pointed out that these spectra are related to a stable nuclear deformation. Investigations of rotational excitations expanded into a very fruitful field of nuclear-structure studies when it became possible to populate high-spin states in fusion-evaporation reactions with heavy projectiles and to perform detailed in-beam γ-ray spectroscopy with large Ge detector arrays.

In recent years it was realised that rotational spectra occur not only in deformed nuclei, but are a more general phenomenon related to spontaneous symmetry breaking of quantum objects [2,3]. The symmetry may be broken by a deformed charge distribution or by anisotropic currents which give rise to symmetry-breaking magnetic moments. Deformed nuclei with axial symmetry can only rotate about an axis perpendicular to the symmetry axis. However, when the axial symmetry is broken, the excitation spectrum may become richer. Bohr and Mottelson [4] suggested about 30 years ago that such triaxial nuclei may show wobbling excitations. They predicted families of rotational bands built on the same intrinsic structure but with some of the collective angular momentum transferred from the axis of largest moment of inertia to the two other principal axes of the triaxial nucleus. Indeed, wobbling is intimately related to stable triaxiality and, thus, its experimental observation is a direct proof of such shapes.

Rotational bands which have been associated with Triaxial Strongly Deformed (TSD) potential-energy minima have been discovered in recent years in several Lu and Hf isotopes, for references see [5]. Lifetime measurements in the Lu isotopes [6,7] show that the TSD bands have indeed very large deformation. The stable triaxiality was confirmed by the discovery of wobbling excitations

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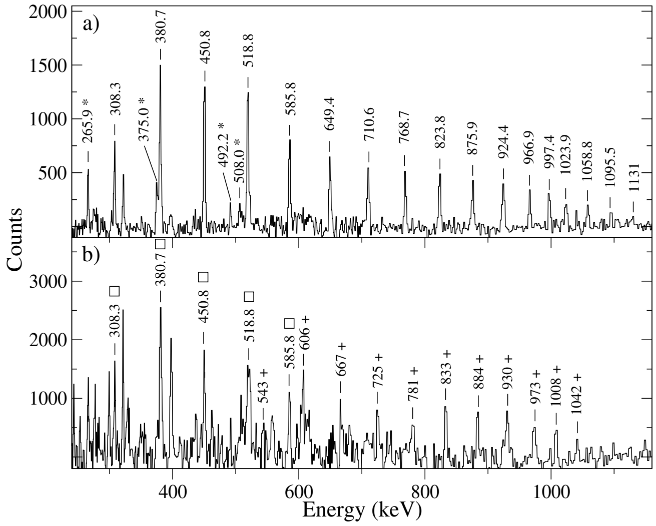

Fig. 1. a) Gamma-ray coincidence spectrum of TSD1 in ^{161}Lu. The spectrum was obtained using triple gates on all transitions of the band (except the 380.7, 450.8 and 518.8 keV transitions). Transitions marked with asterisks correspond to normal-deformed low-lying transitions. Unlabelled peaks are contaminants from neighbouring Yb isotopes. b) Gamma-ray coincidence spectrum of TSD2. The spectrum was obtained using double-gate lists on all transitions indicated by plus signs of TSD2 (except the 606 keV transition). Transitions from TSD1 are marked with a square. Unlabelled peaks are known contaminants.

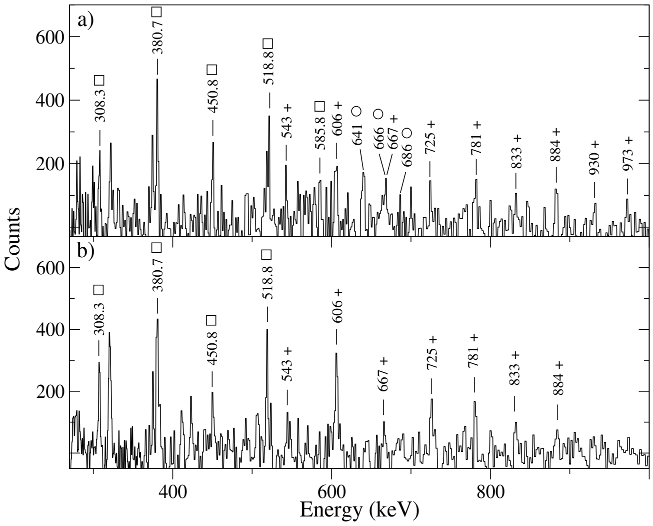

Fig. 2. a) Gamma-ray coincidence spectrum showing inter-band connections between TSD1 and TSD2 in ^{161}Lu. The spectrum was obtained using triple-gate lists. The first list contained the 308.3 and 450.8 keV transitions and the second and third list contained all transitions from TSD2. The same notation as in fig. 1 has been kept for TSD1 and TSD2. The inter-band transitions are marked with a circle. b) Gamma-ray coincidence spectrum showing one connection between TSD1 and TSD2. The spectrum was obtained using double-gate lists. The first contained the 308.3 and 450.8 keV transitions and the second list contained the inter-band 666 keV transition overlapping the 667 keV transition from TSD2. This transition is clearly self-coincident. Unlabelled peaks are contaminants from neighbouring Yb isotopes.

in ^{163}Lu [8–10], ^{165}Lu [11] and ^{167}Lu [12]. The decisive fingerprint for wobbling are inter-band $E2$ transitions between the TSD bands which compete with the strongly enhanced in-band $E2$ transitions [13,14].

In this work, we present the results of a new investigation of ^{161}Lu. In our earlier work [15], only one TSD band, TSD1, was observed. Here, we report on an extension of this band to higher spins and on the discovery of a new TSD band, TSD2. These bands show the same characteristics as the wobbling excitations in the heavier Lu isotopes. In sect. 2 the experimental procedure and results are described and in sect. 3 the evidence for wobbling in ^{161}Lu is discussed.

2 Experimental procedure and results

High-spin states in ^{161}Lu were populated in the reaction ^{139}La(^{28}Si, 6n) at a beam energy of 175 MeV. The beam was provided by the Vivitron accelerator at IReS, Strasbourg. The target consisted of a stack of two self-supporting La foils of $520\,\mu$g/cm^2 thickness each. To prevent oxidation the targets were handled in an argon atmosphere. Gamma-ray coincidences were measured with the EUROBALL spectrometer [16,17]. It comprises 30 conventional, large-volume Ge detectors and 41 composite Ge detectors, all of them surrounded by Compton-suppression shields. Of the composite detectors, 26 are Clovers, each consisting of four Ge crystals and 15 are Clusters, each composed of seven Ge crystals. In addition, an inner ball of 210 BGO crystals was used as multiplicity filter to enhance the detection of high-spin cascades. Coincidence events were written to magnetic tape with a hardware trigger condition of four or more γ-rays detected in the Ge detectors before Compton suppression and 11 or more γ-rays detected in the BGO multiplicity filter. After presorting, which included Compton suppression and add-back of scattered γ-rays, $2.8 \cdot 10^9$ events with a Ge coincidence fold $f \geq 3$ remained for further analysis.

Several three-dimensional matrices (cubes) with different multiplicity and time conditions, as well as a four-dimensional matrix (hypercube) were sorted using the Radware software package [18]. A two-dimensional matrix was also produced using a filtering technique [19] where a filter spectrum containing transitions from the known yrast TSD band (TSD1) in ^{161}Lu [15] was applied. The analysis confirms TSD1 and extends it to higher spins, adding six new transitions to the top of the band. Notice that the energy of the 825.4 keV transition in the earlier work has been corrected to 823.8 keV. A γ-ray coincidence spectrum of this band is shown in the upper panel of fig. 1.

An extensive search through the coincidence cubes and the hypercube revealed a second band with similar

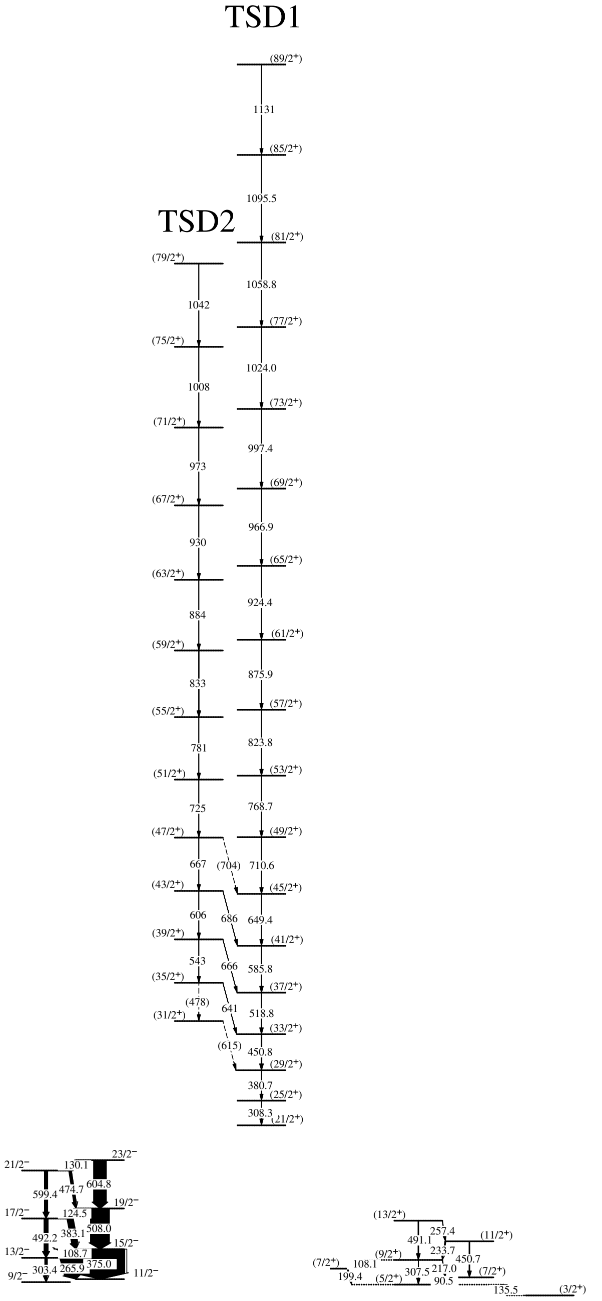

Fig. 3. Partial level scheme of ^{161}Lu showing the two TSD bands and the positive- and negative-parity bands which are seen in coincidence.

characteristics as TSD1. The γ-ray coincidence spectrum of the new band is displayed in fig. 1, lower panel. The intensities of the two bands are estimated to be about 1.4% and 0.6%, respectively, of the population of the ^{161}Lu channel.

The new band, TSD2, is in coincidence with the γ-rays de-exciting the low-spin states of the previously known band, TSD1. Figure 2 shows coincidence spectra with energy gates set, in the upper panel, on high-spin transitions of TSD2 and on low-spin transitions of TSD1 and, in the lower panel, on low-spin transitions of TSD1 and on the 666 keV inter-band transition. The bands are clearly in mutual coincidence. Weak inter-band transitions, from TSD2 to TSD1, with energies between 641 and 686 keV are observed. Moreover, the 666 keV transition is clearly self-coincident. A partial level scheme of ^{161}Lu is displayed in fig. 3. It shows the two TSD bands and normal-deformed (ND) states populated in their decay.

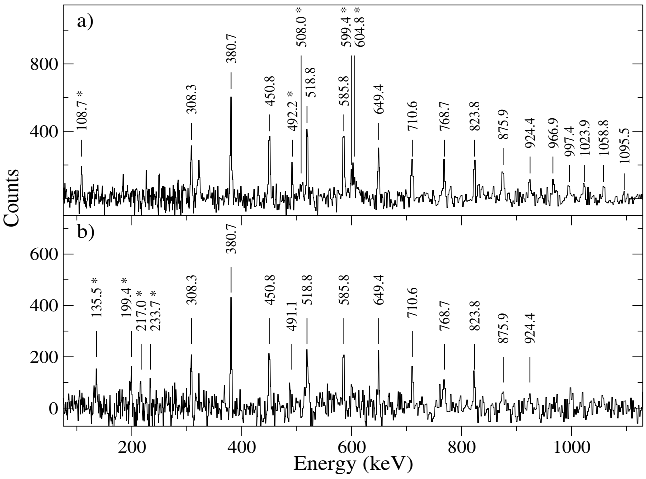

Fig. 4. a) Gamma-ray spectrum of TSD1 in coincidence with the negative-parity ND band. The spectrum was obtained using triple-gate lists. The first list contained the ND 265.9 keV transition and the second and third lists contained all transitions from TSD1. The ND transitions are marked with an asterisk. b) Gamma-ray spectrum of TSD1 in coincidence with a presumably positive-parity ND band. The spectrum was obtained using triple-gate lists. The first list contained the ND 135.5 and 199.4 keV transitions and the second and third lists contained all transitions from TSD1. The ND transitions are marked with an asterisk.

Transitions linking the TSD bands to low-lying levels of the ND level scheme of ^{161}Lu could not be firmly established. However, the assignment to this nucleus is unambiguous, as several ND transitions are observed in coincidence (see, *e.g.*, fig. 1a). The spin assignment shown in fig. 3 is based on the close similarity of transition energies of TSD1 with those determined for TSD1 in ^{163}Lu [10]. Two different decay pathways have been identified (see fig. 4). One main decay path leads to the negative-parity band [20] shown in fig. 3, at least up to spin $23/2^-$. This supports the spin assignment for the TSD bands, since the main decay-out starts at spin $(25/2^+)$. Another weaker path leads to a presumably positive-parity band [21,22], which is observed up to spin $(13/2^+)$. In this case the decay probably proceeds in several steps.

3 Evidence for wobbling excitation

The wobbling mode is a unique fingerprint of stable nuclear triaxiality [4]. It gives rise to families of rotational bands which are built on the same intrinsic structure, but have different tilt angles of the collective-angular-

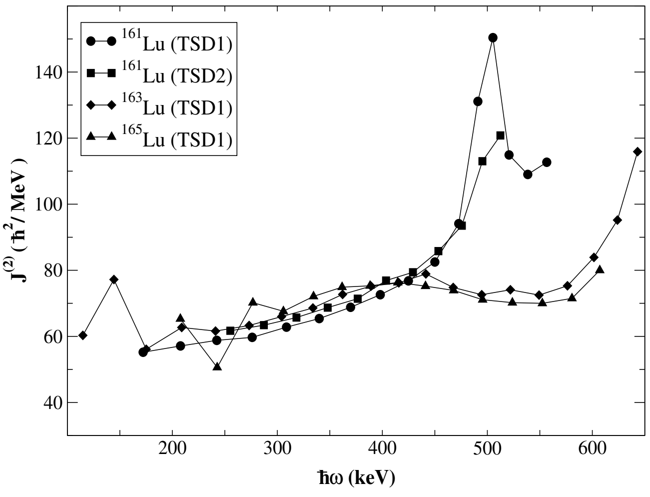

Fig. 5. Dynamic moments of inertia, $J^{(2)}$, as a function of rotational frequency, $\hbar\omega$, for the two TSD bands in ^{161}Lu in comparison with the yrast TSD bands in ^{163}Lu and ^{165}Lu.

momentum vector with respect to the principal axis of the nucleus that carries the main component. The lowest-energy band, the band with wobbling-phonon quantum number $n_{\mathrm{w}} = 0$, corresponds to the rotation about the axis of the largest moment of inertia. The bands with $n_{\mathrm{w}} = 1, 2, \ldots$ are associated with larger and larger tilt angles with respect to that axis [4].

The first case of nuclear wobbling was established in ^{163}Lu [8–10]. In this case, a family of three bands was discovered with strong inter-band transitions which could only be explained within the framework of the wobbling picture [13,14]. Very similar bands were subsequently found in ^{165}Lu [11] and ^{167}Lu [12]. In these cases the bands are weaker and the wobbling interpretation was, to a certain extent, based on the close analogy to ^{163}Lu. For the present case, ^{161}Lu, we are also using the analogy to ^{163}Lu.

The dynamic moments of inertia, $J^{(2)}$, of the two bands in ^{161}Lu are compared with those of the yrast TSD bands (TSD1) in ^{163}Lu and ^{165}Lu in fig. 5. The two ^{161}Lu bands have very similar moments of inertia; they are also similar to the ones of the heavier isotopes up to a frequency of about 450 keV. Above this frequency, the ^{161}Lu bands show a sharp rise in $J^{(2)}$ indicative of an alignment. The heavier isotopes do not show this pronounced alignment around 500 keV. Their moments of inertia rise sharply only above a frequency of 600 keV. The two ^{161}Lu bands show the alignment in the same frequency range, as expected for bands with the same intrinsic structure.

The TSD bands in the heavier Lu isotopes have been associated with local large-deformation minima with a pronounced triaxiality (ϵ_2, γ) = (0.38, 20°) found in Total-Routhian-Surface (TRS) calculations [8–12]. TRS calculations performed with the Ultimate Cranker (UC) code [23,24] for ^{161}Lu show similar minima. An example of the results of such calculations is displayed in fig. 6. Based on the similarity of the TSD bands in the Lu isotopes, we also associate the two bands in ^{161}Lu with the TSD minimum at (0.38, 20°) seen in the calculations. The local minimum at positive γ-deformation is somewhat deeper than that at negative γ. Particle-rotor calculations [13] show that wobbling solutions are only obtained for $\gamma > 0$.

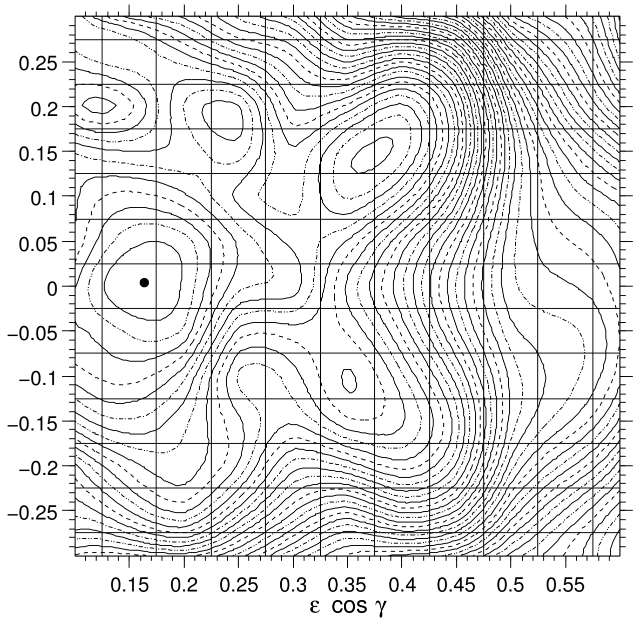

Fig. 6. Total Routhian Surface for ^{161}Lu calculated using the Ultimate Cranker code [23,24] for parity and signature $(\pi, \alpha) = (+, 1/2)$ and spin $I = 57/2\ \hbar$. The energy difference between the contour lines is 0.2 MeV.

The close similarity of the dynamic moments of inertia of the two TSD bands in ^{161}Lu, see fig. 5, is already a first indication that they are built on the same intrinsic structure. Their alignments further support this assumption. The aligned angular momentum, i_x, of the two bands is compared with those of the heavier isotopes, ^{163}Lu and ^{165}Lu, in fig. 7. In all cases the $n_{\mathrm{w}} = 1$ excited wobbling band shows a slightly higher ($\sim 0.5\ \hbar$) alignment than the $n_{\mathrm{w}} = 0$ band. With the adopted spin assignments for TSD1 and 2 the alignments of the ^{161}Lu bands are almost identical to those of the heavier isotopes near the bottom of the bands. At higher frequencies the ^{161}Lu alignments are $\sim 0.5\ \hbar$ lower. Above a frequency of ~ 450 keV, the ^{161}Lu bands show a strong increase in i_x of at least 6 $\hbar$. Inspection of the calculated quasiparticle Routhian diagrams suggests that this gain in alignment is probably caused by $i_{13/2}$ or $j_{15/2}$ quasineutrons. Other quasiparticles give a smaller alignment. It should be noted, however, that the UC calculations predict the neutron $i_{13/2}$ alignment at lower frequencies. In the heavier isotopes, the general rise observed in the alignment has been suggested to be caused by a gradual alignment of $i_{13/2}$ quasineutrons [25]. Possibly the initial rise in alignment below a frequency of 450 keV, see fig. 7, is caused by $i_{13/2}$ quasineutrons also in ^{161}Lu. The strong alignment at higher frequencies may then be caused by $j_{15/2}$ quasineutrons. In the UC calculations a hybrid crossing of a $j_{15/2}$ with another negative-parity orbital comes down in frequency to ~ 500 keV if a small negative ε_4 is introduced.

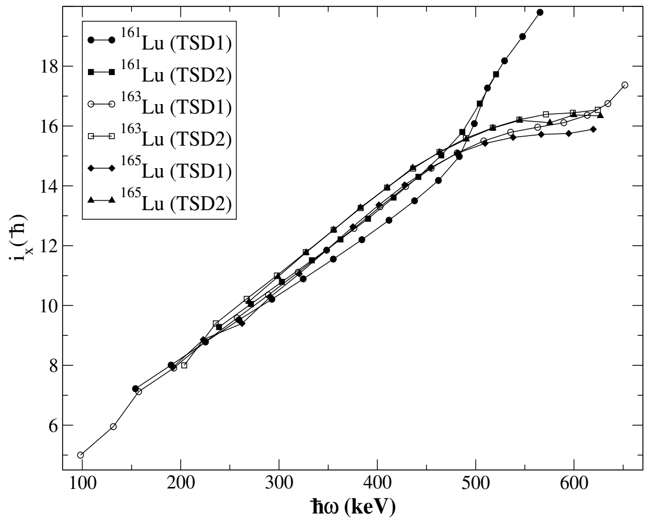

Fig. 7. Aligned angular momenta, i_x, relative to a reference as a function of rotational frequency, $\hbar\omega$, for the two TSD bands in ^{161}Lu in comparison with the two, yrast and first-excited, TSD bands in ^{163}Lu and ^{165}Lu. The chosen reference was $i_{\text{ref}} = \Im_0\omega + \Im_1\omega^3$ with $\Im_0 = 30\ \hbar^2\text{MeV}^{-1}$ and $\Im_1 = 40\ \hbar^4\text{MeV}^{-3}$.

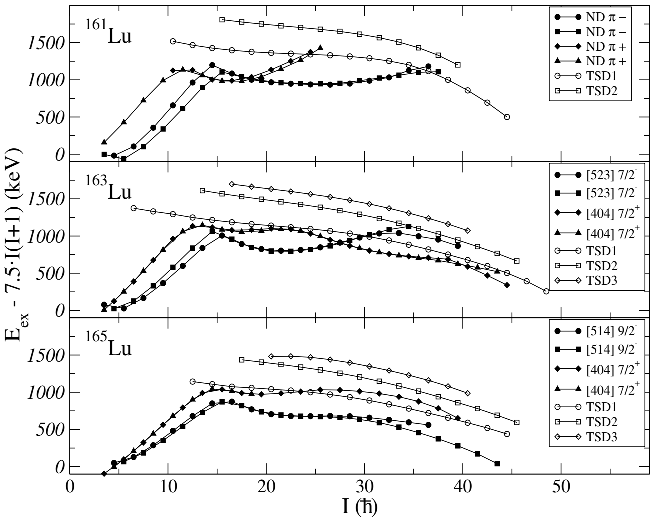

Fig. 8. Excitation energies as a function of spin of TSD bands in ^{161}Lu [22], ^{163}Lu [9] and ^{165}Lu [27], respectively. Spins and excitation energies of the TSD bands in ^{161}Lu are tentative.

Such a small change in shape may explain the difference to the heavier isotopes. An alternative explanation might be a second $i_{13/2}$ quasineutron crossing with less interaction in ^{161}Lu compared with the heavier isotopes. With its neutron number $N = 90$, ^{161}Lu lies near the middle of the $i_{13/2}$ neutron subshell [26] where the behaviour depends strongly on γ-deformation and a small variation may produce a drastic change. A small difference in shape may also explain the 0.5 $\hbar$ difference observed in the alignments compared to the heavier isotopes since this is sensitive to the choice of the reference [25].

The excitation energies of selected ND and the TSD bands in ^{161,163,165}Lu are compared in fig. 8. Unfortunately the energies and spins of the TSD bands in ^{161}Lu are not firmly established experimentally. As mentioned above, the close similarity to the TSD bands in the heavier isotopes supports the tentative spin assignment shown in the level scheme of fig. 3. The approximate excitation energy can be estimated considering the energies relative to the ND states. As can be seen, the relative position of the TSD bands is similar in the three isotopes. In particular, the excitation energy of TSD2 with respect to TSD1 is around 650 keV in all three isotopes.

Two different decay-out scenarios have been observed in the Lu isotopes [5]. The mechanisms depend on the excitation energy of the TSD minimum and the height of the barrier between the TSD and the ND minima. Mixing of the TSD and ND states may occur when they come close in energy, which is the case in ^{163}Lu, ^{165}Lu and ^{167}Lu [9,11, 12], leading to the decay. Alternatively, the bands can decay by stretched and unstretched $E1$ and $E2$ transitions, as observed in ^{164}Lu [28]. In the case of ^{161}Lu, no decay-out has been firmly established but the strongest decay is definitely to the negative-parity band with which it cannot mix. Mixing with positive-parity states is also unlikely considering the smooth behaviour of the dynamic moment of inertia, $J^{(2)}$, at low frequencies, see fig. 5. The unobserved decay is probably fragmented across several unresolved transitions and should resemble the ^{164}Lu scenario.

The systematic behaviour of the TSD bands in the odd-A Lu isotopes makes it very likely that bands TSD1 and TSD2 in ^{161}Lu are the $n_{\text{w}} = 0$ and 1 wobbling excitations. Unfortunately, the electromagnetic properties of the $\Delta I = 1$ transitions between the bands, which would be the final proof, could not be determined in our experiment. Since the population of the bands is considerably weaker than found for TSD bands in ^{163,165}Lu, it is not surprising that a second phonon wobbling excitation, as established in those nuclei, could not be found in ^{161}Lu.

There is evidence for wobbling in the odd-mass Lu isotopes, but not in the odd-odd Lu isotopes and also not in the even-even Hf isotopes [29–32] where TSD bands are known. This may be related to the presence of the aligned $i_{13/2}$ proton in the odd Lu isotopes which stabilises the triaxial shape. However, a full understanding may require further theoretical investigations. Furthermore, experiments with better statistical accuracy may be required as the wobbling bands in the other isotopes may lie at higher excitation energies and, therefore, would be populated with less intensity.

4 Summary

In summary, the previously known band TSD1 in ^{161}Lu was extended to considerably higher spins and a new TSD band, TSD2, was discovered, which decays into TSD1. The systematic behaviour of the TSD bands in the odd-mass Lu isotopes strongly suggests an interpretation of the TSD bands in ^{161}Lu as wobbling excitations with wobbling-phonon quantum number $n_{\text{w}} = 0$ and 1.

The authors are grateful to the technical staff at IReS for running the Vivitron and the EUROBALL spectrometer array. The work was supported by the EU (contract No. HPRI-CT-1999-00078), by BMBF Germany (contract No. 06 BN 907), by the Danish Science Foundation, by the UK Engineering and Physical Sciences Research Council, by the German Academic Exchange Service (DAAD), by DFG and by the A. v. Humboldt foundation.

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